

Bearings:-

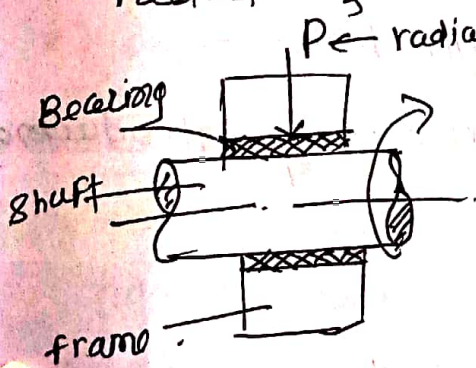
- Bearing is a mechanical element that permits relative motion between two parts, such as the shaft & housing, with minimum friction.

function:-

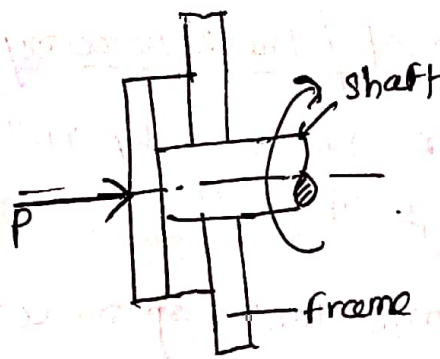
- i) ensure free rotation of shaft with min. friction.
- ii) support & hold shaft in correct position
- iii) Bearing take up forces that act on shaft & transmit it to frame or foundation.

Classification

A) Based on direction of force that act on bearing radial & thrust bearing



Radial bearing



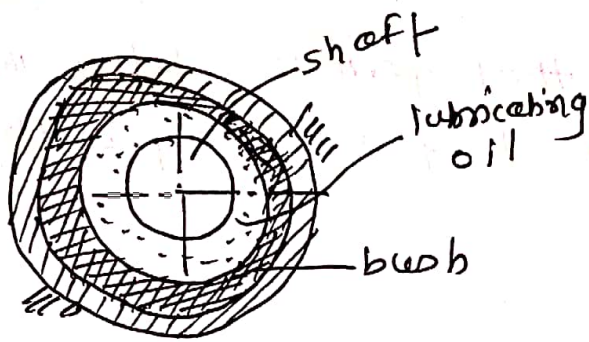
Thrust bearing

Radial bearing - support the load, which is perpendicular to the axis of shaft

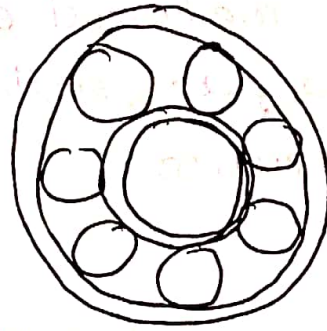
Thrust bearing - support load, which acts along axis of the shaft.

B) Based on friction betⁿ shaft and bearing

- i) sliding contact / Journal / sleeve / plain bearing
- ii) Rolling contact / Ball bearing / Antifriction.



sliding contact bearing



Rolling contact Bearing

i) sliding contact Bearing :-

- surface of shaft slide over the surface of bush
- In order to reduce friction, two surfaces are separated by film of lubricating oil.
- Bush - white metal / bronze.

ii) Rolling contact / Ball Bearing / Antifriction :-

- sliding friction replaced by rolling friction by introducing rolling elements like ball.

Application i) m/c tool spindles

ii) automobile front & rear axle.

iii) gear boxes

iv) small electric motor

v) rope sheaves, hoisting drum.

- Rolling contact bearing consist of four parts - inner & outer race, rolling element like ball, roller or needle & cage. which hold a rolling element together & space them evenly around periphery of shaft.
- Depending on type of ~~the~~ rolling element, bearing is classified as ball bearing, cylindrical, roller bearing, taper roller bearing, needle bearing.

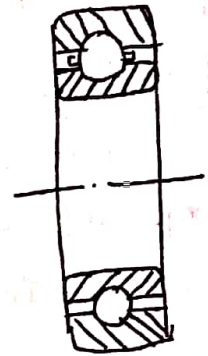
Types of Rolling Contact Bearing :-

① Deep Groove Ball bearing :-

- radius of ball is slightly lesser than radius of curvature of grooves in the races.
- point contact (without sliding)

Advantages :

- high load carrying capacity due to large size balls
- take load in radial & axial direction
- point contact \rightarrow less friction - low temp. rise \rightarrow excellent performance in high speed application.
- less noise due to point contact
- available with bore dia. of few mm to 400 mm.



Deep groove Ball bearing

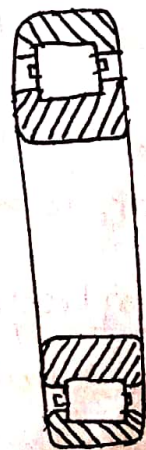
Disadvantages :-

- not self aligning (so accurate alignment of shaft & housing bore required).
- poor rigidity (pt. contact \rightarrow no line contact) unsuitable for m/c tool spindles.

② Cylindrical Roller Bearing :-

- Maximum load carrying capacity required in given space, is achieved through ~~replacing~~ replacement of pt. contact by line contact.

- Instead of ball, short roller can be used that are positioned & guided by cage.



Advantages:

- i) high radial load carrying capacity (line contact)
- ii) more rigid than ball bearing
- iii) coeff. of friction is low & frictional loss is less in high speed application.

Disadvantages:-

- i) can not take thrust load.
- ii) not self aligning & can not tolerate misalignment
- iii) generate more noise.

⑥ Angular contact Bearing :-

- In angular contact bearing, the grooves in inner and outer races are so shaped that the line of reaction at the contact between balls and races makes an angle with the axis of bearing

- These reaction has two component - radial & thrust.

- angular contact bearing can take radial & thrust load.

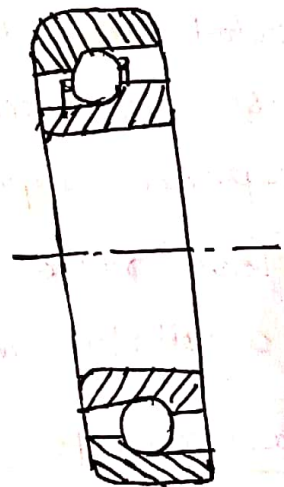
- It is often used in pair either side by side or at opposite ends of the shaft in order to take thrust load in both direction.

- Bearing assembled with specific magnitude of preload.

Advantages:-

i) take both radial and thrust load

ii) carry relatively large load (axial & radial) than ball bearing due to large no. of ball insertion permitted as one side of outer race is cut away.



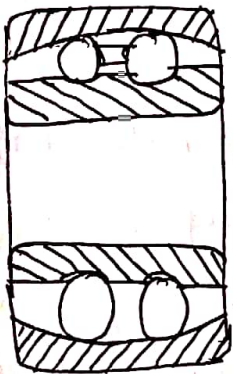
Angular contact Ball bearing

Disadvantages :-

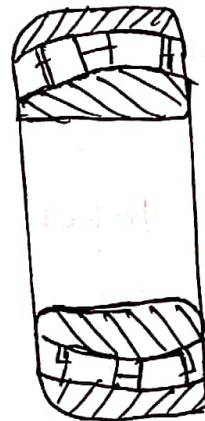
- i) two bearings required to take thrust load in both direction.
- ii) mounted without axial play.
- iii) requires initial pre-loading.

④ Self aligning Bearing :-

self aligning ball bearing



self aligning spherical roller bearing



- Two rows of ball rolls on common spherical surface in outer race.
- an assembly of shaft, inner race & ball with cage can freely roll & adjust itself to angular misalignment

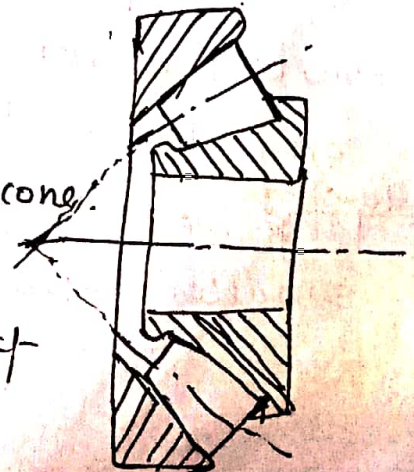
- Two rows of spherical roller run on common spherical surf in outer race

- carry relatively high load

Application :- agricultural machinery, railway axle boxes.

⑤ Taper Roller Bearing

- Rolling element - in the form of frustum of cone.
- Arrangement of rolling element is such a way that axis of each frustum intersect at common apex point on axis of bearing



- carry both radial and axial load.
- used in pair to balance thrust component
- separable construction - outer ring (cup) & inner ring (cone)
Cup is separable from remainder assembly.

Advantages:-

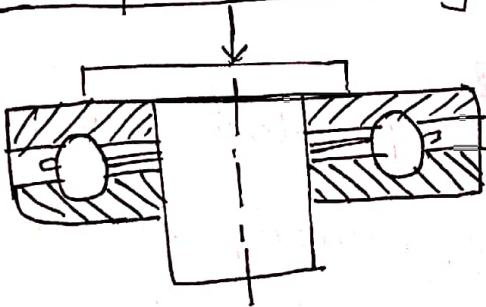
- take heavy radial & thrust load
- more rigid
- easily assembled & dismantled due to separable parts.

Disadvantages:-

- need to use two taper roller bearings for balancing thrust
- need to adjust pre-load for coinciding apex of cone with common apex of rolling element.
- can not tolerate misalignment betⁿ axis of shaft & housing bore.
- costly.

Application :- car & trucks, propeller shaft & differential.

⑥ Thrust Ball bearing:-



- row of balls running betⁿ two rings (shaft ring & housing ring)
- carry thrust load in one direction

Advantage
- large ball - high thrust load carrying capacity. in smaller space

Dis Advantages:-

- i) can not take radial load
- ii) can not tolerate misalignment
- iii) satisfactory performance at low & medium speed.
poor performance - at high speed \Rightarrow bcoz balls are subjected to centrifugal force & gyroscopic couple.
- iv) won't give good performance with horizontal shaft
- v) Thrust ball bearing requires continuous pressure applied by spring to hold the rings together.

* Material of Rolling contact Bearing :-

- Balls, inner race & outer race \rightarrow high carbon chromium steel
(SAE 52100 or AISI 5210)

1% C, 1.5% Cr

balls & races \rightarrow through harden \rightarrow 58 Rockwell C

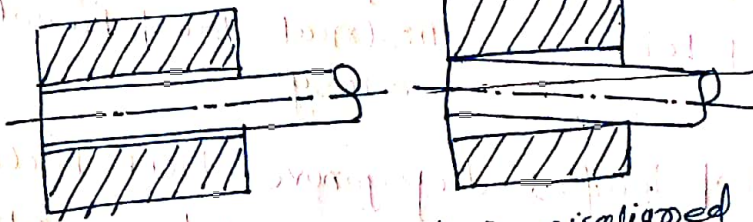
- Cage - stamping of low carbon steel.

- Rollers :- case hardened steel (AISI 3310, 4620, 8620)
RC-58 \rightarrow case carburized.

Rolling contact Bearing

* Principle of self Aligning Bearing :-

- self aligning ball bearing & spherical roller bearing
- tolerate small amount of misalignment between the axes of the shaft and the bearing.
- misalignment - due to deflection of shaft under load or due to tolerance of individual component.



shaft aligned with bearing

shaft misaligned with bearing

- Deflected shaft exerts pressure at edges of bearing
- edge pressure is dangerous & may result in undue wear & breakdown of oil film.

↑ selection of Bearing type : Guidelines

- i) low & medium radial load → ball bearing
heavy load & large shaft dia → roller bearing
- ii) misalignment betⁿ axes of shaft & housing → self aligning
is likely to exist → ball or spherical roller bearing
- iii) Medium thrust load → Thrust ball bearing
heavy thrust load → cylindrical thrust roller bearing
- iv) radial & thrust load → deep groove ball bearing, angular contact bearing, spherical roller bearing.
- v) high speed application → deep groove ball bearing, angular contact, cylindrical roller
- vi) Rigidity → Double row cylindrical roller, Taper roller bearing
- vii) Noise → deep groove ball bearing

Static load carrying capacity :-

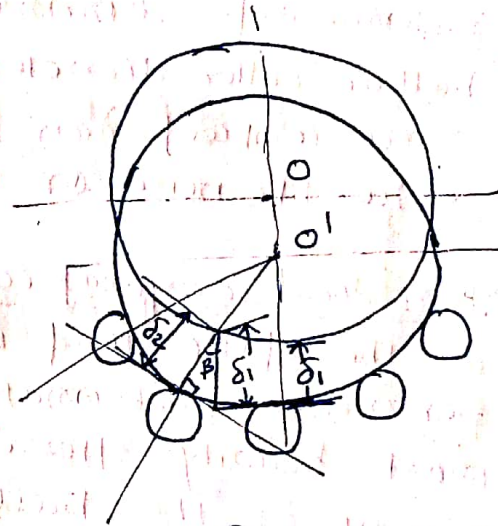
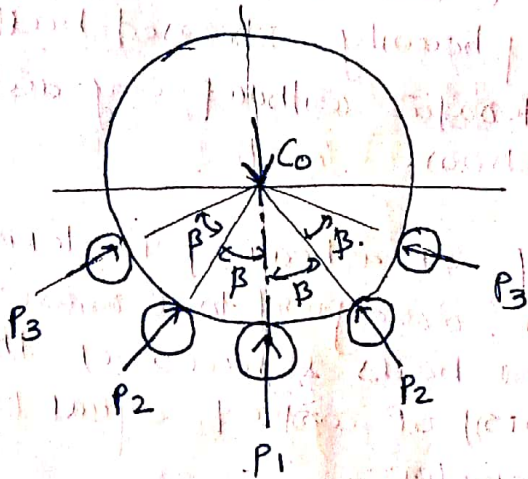
- Static load is defined as the load acting the bearing when the shaft is stationary.
- It produces permanent deformation in ball & races, which increases with increasing load.
- permissible static load corresponds to permissible permanent deformation (1×10^{-4} or 0.0001) of ball or roller diameter at heavily stressed ball & race contact can be tolerated without any disturbance like noise or vibration.
- Static load carrying capacity of a bearing is defined as the static load which corresponds to a ~~max~~ total permanent deformation of balls & races at the most heavily stressed point of contact equal to 0.0001 of the ball diameter.

↳ Where conditions of friction, noise & smoothness are not critical, a much higher permanent deformation can be tolerated & consequently static loads up to four times of static load carrying capacity may be permissible.

* Stribeck's Equation: - gives static load carrying capacity of bearing. -

Assumption: -

- i) races are rigid & retain their circular shape
- ii) balls are equally spaced.
- iii) balls in upper half do not support any load.



(a)

(b)

→ Fig (a) shows forces on inner race through rolling element which support static load C_o .

- Equilibrium of vertical forces in vertical plane.

$$C_o = P_1 + 2P_2 \cos \beta + 2P_3 \cos(2\beta) + \dots \quad \text{--- (1)}$$

- As races are rigid, balls are deformed.

- δ_1 is the deformation at most heavily stressed Ball No. 1.

- Due to deformation, the inner race is deflected with respect to outer race through δ_1

- Centre of inner race moves from O to O' through distance δ_1 without changing its shape.

- $\delta_1, \delta_2 \dots$ are radial deflection at respective ball.

$$\cos \beta = \frac{\delta_2}{\delta_1} \quad \text{--- (2)}$$

According to Hertz's eqⁿ, the relationship betⁿ load & deflection at each ball is given by

$$\delta \propto P^{(2/3)}$$

$$\begin{aligned} \delta_1 &= C_0 P_1^{2/3} \\ \delta_2 &= C_0 P_2^{2/3} \end{aligned} \quad \therefore \frac{\delta_2}{\delta_1} = \left(\frac{P_2}{P_1} \right)^{2/3} = \cos \beta \quad \text{--- (3)}$$

$$\frac{P_2}{P_1} = (\cos \beta)^{3/2}$$

$$P_2 = P_1 (\cos \beta)^{3/2}$$

Similarly $P_3 = P_1 (\cos 2\beta)^{3/2}$

substituting P_2 & P_3 ... in (1)

$$\begin{aligned} C_0 &= P_1 + 2 [P_1 (\cos \beta)^{3/2}] \cos \beta + 2 [P_1 (\cos 2\beta)^{3/2}] \cos 2\beta \\ &\quad + \dots \\ &= P_1 [1 + 2 (\cos \beta)^{5/2} + 2 (\cos 2\beta)^{5/2} + \dots] \end{aligned}$$

$$[C_0 = P_1 M] \quad \text{--- (4)}$$

where

$$M = [1 + 2 (\cos \beta)^{5/2} + 2 (\cos 2\beta)^{5/2} + \dots]$$

If Z is the no. of balls

$$\beta = \frac{360}{Z}$$

Z	8	10	12	15
M	1.84	2.28	2.75	3.47
Z/M	4.35	4.38	4.36	4.37

$(Z/M) = \text{almost constant}$

Stribeck suggested the value of $(Z/\eta) = 5$

$$\frac{Z}{\eta} = 5 \Rightarrow \eta = \left(\frac{1}{5}\right)Z$$

substituting $\eta = (1/5)Z$ into (4)

$$C_0 = \frac{1}{5} Z P_1 \quad \text{-----} \quad (5)$$

From experimental evidence, it is found that the force P_1 required to produce a given permanent deformation of ball is given by

$$P_1 = kd^2 \quad \text{-----} \quad (6)$$

d = diameter of ball

k = factor depends upon radii of curvature at point of contact & on models of elasticity of material

from (5) & (6)

$$C_0 = \frac{1}{5} Z \cdot kd^2$$

$$C_0 = \frac{kd^2 Z}{5}$$

This equation is known as Stribeck's eqⁿ.

* Dynamic load carrying capacity :-

- The life of bearing is limited by fatigue failure at the surfaces of balls & races.
- The life of an individual ball bearing is defined as the no. of revolution (or hours of service at some given speed) which bearing runs before the evidence of first fatigue crack in balls or race.
- Life of single bearing is difficult to predict so life in terms of statistical average performance of group of bearing is defined. based on two criteria.

- Average life of group of bearing
- Life which 90% bearing will reach. → widely used

- The rating life of a group of apparently identical ball bearing is defined as the no. of revolutions that 90% of the bearings will complete before the first evidence of fatigue crack.

rating life — min. life / catalogue life / L_{10} life / B_{10} life

- Life of individual bearing may be different from rating life
- Statistically life with 50% of group of bearings will complete is approximately five times rating life.
This means for majority of bearing, the actual life is considerably more than rated life

- Dynamic load carrying capacity :-

It is defined as the radial load in radial bearing (thrust load in thrust bearing) that can be carried for a minimum life of one million revolution.

- Dynamic load carrying capacity is based on assumption that inner race is rotating & outer race is stationary

Equivalent bearing load:-

In actual applications, forces acting on bearing has two components - radial & thrust

- It is therefore necessary to convert two components into single hypothetical load, fulfilling conditions applied to dynamic load.

- This hypothetical load can be compared with dynamic load capacity.

→ The equivalent dynamic load is defined as the constant radial load in radial bearing (or thrust load in thrust bearing) which is applied to the bearing would give same life as that which the bearing will attain under actual condition of forces.

- Expression for equivalent dynamic load

$$P = X Y F_r + Y F_a$$

P = equivalent dynamic load (N)

F_r = radial load

F_a = axial load

X = radial factor

Y = thrust factor

V = race-rotation factor

$V=1$ → inner race rotating
outer stationary

$V=1.2$ = inner race stationary
outer rotating

When the bearing is subjected to pure radial load F_r

$$P = F_r$$

When bearing is subjected to pure thrust load F_a ,

$$P = F_a$$

Load Life Relationship :-

The relationship betⁿ dynamic load carrying capacity, equivalent dynamic load & bearing life is given by

$$L_{10} = (C/P)^p$$

where L_{10} = rated bearing life (in million revolutions)

C = dynamic load capacity (N)

$p = 3$ — for ball bearing

$p = 10/3$ → roller bearing

$$C = P (L_{10})^{1/p}$$

The relation between life in million revolutions & life in working hours is given by

$$L_{10} = \frac{6000 \cdot L_{10hr}}{10^6}$$

Example :-

- ① In a particular application the radial load acting on a ball bearing is 5 kN and expected life for 90% of the bearing is 8000 h. Calculate the dynamic load carrying capacity of bearing when shaft rotates at 1450 rpm.

$$\begin{aligned} \rightarrow L_{10} &= 696 \text{ million rev.} & L_{10} &= \frac{60 \pi L_{10} \text{ hr}}{10^6} \\ C &= 44310.48 \text{ N.} & C &= P(L_{10})^{1/3} \end{aligned}$$

- ② A taper roller bearing has dynamic capacity of 26 kN. The desired life for 90% of the bearing is 8000 h and the speed is 300 rpm. Calculate the equivalent radial load that the bearing can carry.

$$\begin{aligned} \rightarrow L_{10} &= 144 \text{ million rev.} \\ P &= 5854.16 \text{ N.} \\ F_r &= P = 5854.16 \text{ N.} \end{aligned}$$

$$L_{10} = \frac{60 \pi L_{10} \text{ hr}}{10^6} = \frac{60 \times 300 \times 8000}{10^6} = 144 \text{ million rev}$$

$$C = P(L_{10})^{0.3}$$

$$P = \frac{C}{(L_{10})^{0.3}} = \frac{26000}{(144)^{1/3}} = 5854.16 \text{ N.}$$

* selection of Bearing life:-

- While selecting the proper size of bearing, it is necessary to specify the expected life of the bearing for given application (from past experience)

— for all vehicle, speed of rotation is not constant so desired life is expressed in terms of millions of revolution.

Bearing life for wheel application

Wheel application	Life in (million rev.)
Automobile car	50
Truck	100
Trolley car	500
Rail-Road cars	1000

— In some applications, speed of rotation is constant & desired life is expressed in terms of hrs

i) M/c used intermittently	4000 - 8000 h
ii) M/c used for 8 hrs/day	12000 - 20000 h
iii) M/c used continuously (24 hrs/day)	40,000 - 60,000 h

Load factor:-

- Forces acting on bearing are calculated by considering eqⁿ of forces in vertical & horizontal planes. These elementary eqⁿ do not take into account the effect of dynamic load.
- The forces determined by these equations are multiplied by load factors to determine dynamic load carrying capacity of bearing.
- Load factors used in application involving gears, chain, belt drives

Types of drive

Load Factors

(A)

Gear drives

- i) Rotating m/c free from impact (ex. electric motor)

1.2 - 1.4

- ii) Reciprocating m/c like IC engine & compressors

1.4 - 1.7

- iii) Impact m/c like hammer

2.5 - 3.5

(B) Belt drives

- i) V-belt

2

- ii) Single ply leather belt

3

- iii) Double ply leather belt

3.5

(C) Chain drives

1.5

* Selection of Bearing From Manufacturer's Catalogue :- procedure for selection of bearing.

① Calculate F_r & F_a
Determine dia. of shaft where bearing is to fitted.

② select type of bearing.

③ Determine values of X & Y from catalogue

X & Y depends on $\left(\frac{F_a}{F_r}\right)$ & $\left(\frac{F_a}{C_0}\right)$ where C_0 = static load capacity.

- selection of bearing is therefore by trial error.
- start selection of bearing with light series first & medium & heavy.

④ Calculate equivalent dynamic load

$$P = X F_r + Y F_a$$

⑤ Make decision about expected bearing life & express life in L_{10} in million revolution.

⑥ Calculate dynamic load capacity

$$C = P (L_{10})^{1/3}$$

⑦ check whether selected bearing of series has required dynamic capacity. if not, select the bearing of next series & go back to 3rd step.

⊕ Example

① A single row deep groove ball bearing is subjected to a pure radial force of 3 kN from a shaft that rotates at 600 rpm. The expected life L_{10h} of the bearing is 30000 h. The minimum acceptable diameter of the shaft is 40 mm. Select a suitable ball bearing for this application

→ $P = F_r = 3000 \text{ N}$
 $L_{10} = 1080 \text{ million rev.}$
 $C = 30779.57 \text{ N.}$

6208 6308

② A single row deep groove ball bearing is subjected to a radial force of 8 kN and a thrust force of 3 kN. The shaft rotates at 1200 rpm. The expected life L_{10h} of the bearing is 20,000 h. The minimum acceptable diameter of the shaft is 75 mm. Select the suitable ball bearing for this application

→ Given :- $F_r = 8 \text{ kN}$ $L_{10hr} = 20,000 \text{ hr.}$
 $F_a = 3 \text{ kN.}$ $\phi = \cancel{20,000} 75 \text{ mm.}$
 $n = 1200 \text{ rpm}$

$\frac{f_r}{F_r} = \frac{3}{8} = 0.375$ $\frac{f_a}{C_0} = \frac{3 \times 10^3}{9800} = 0.3061$

e corresponding to $\frac{f_a}{C_0} = 0.3061$ is

$$\frac{n_2 - n_1}{n - n_1} = \frac{y_2 - y_1}{y - y_1} \quad \therefore \frac{0.5 - 0.25}{0.3061 - 0.25} = \frac{0.44 - 0.37}{y - 0.37}$$

$y = 0.3857 = \underline{\underline{e}}$

$\underline{\underline{e}} \geq \frac{f_a}{F_r} \quad X = 1, Y = 0$

$P = X F_r + Y F_a = 1(8) + 0(3) = 8 \text{ kN.}$

$L_{10} = \frac{60n L_{10hr}}{10^6} = \frac{60 \times 1200 \times 20000}{10^6} = 1440 \text{ million rev}$

$$C = P(L_0)^{1/3}$$

$$C = 8 \times 10^3 (1440)^{1/3} = \underline{\underline{90,339.46 \text{ N}}}$$

As $C_0 = 9800$ corresponding $C = 12500$ so the selected bearing is not suitable.

Assuming $C_0 = 72000$.

$$\frac{f_a}{f_r} = 0.375 \quad \frac{F_a}{C_0} = \frac{3000}{72000} = 0.04167$$

$$e = 0.24 = \frac{f_a}{f_r} > e$$

By linear interpolation

$$Y = 11.8 - \frac{(11.8 - 11.6)}{0.07 - 0.04} (0.04167 - 0.04) = 11.79$$

$$X = 0.56$$

$$P = X f_r + Y f_a$$

$$= 0.56(8000) + 11.79(3000)$$

$$= 9850 \text{ N}$$

$$C = P(L_0)^{1/3} = 9850 (1440)^{1/3}$$

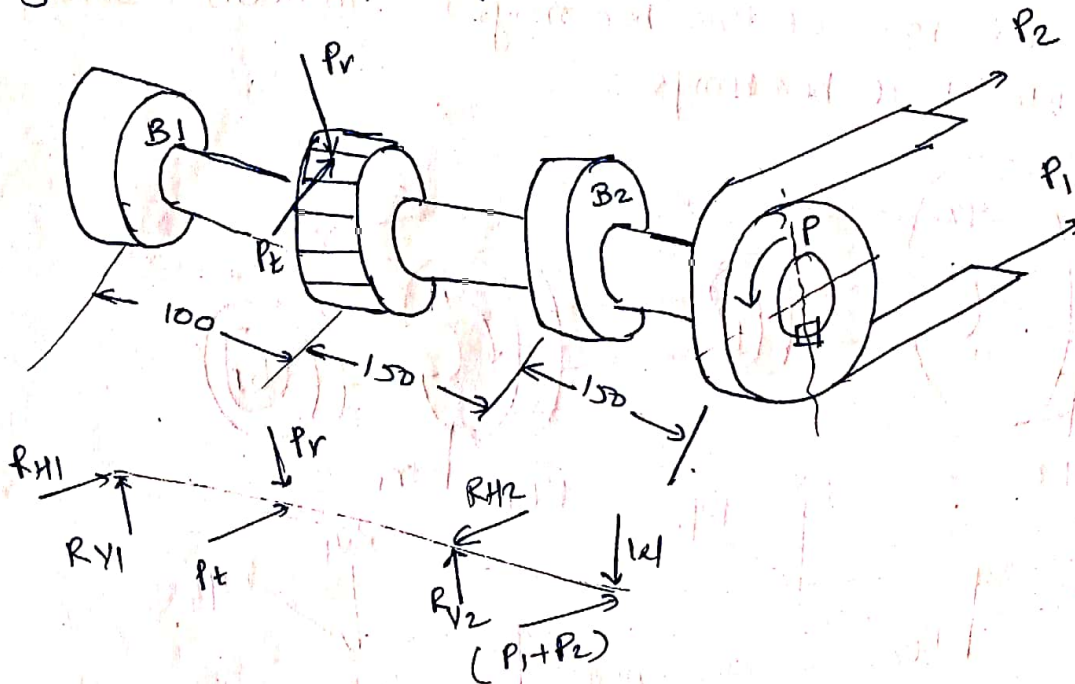
$$= \underline{\underline{111230.46 \text{ N}}}$$

As selected bearing has $C = 112000$, which is ~~gr~~ more than calculated $C = 111230 \text{ N}$.

So the selected bearing is suitable for the

application i.e. 6315

③ A transmission shaft rotating at 720 rpm and transmitting power from the pulley P to the spur gear G is shown in fig. The belt tensions and the gear tooth forces are as follows $P_1 = 498 \text{ N}$, $P_2 = 166 \text{ N}$, $P_t = 497 \text{ N}$, $P_r = 181 \text{ N}$. The weight of the pulley is 100 N . The diameter of the shaft at bearings B_1 & B_2 is 10 mm and 20 mm resp. The load factor is 2.5 and the expected life for 90% of the bearings is 8000 h. Select single deep groove ball bearings at B_1 & B_2 .



$$R_{V1} = 232.4 \text{ N}$$

$$R_{H1} = 100.2 \text{ N}$$

$$R_1 = \sqrt{R_{V1}^2 + R_{H1}^2}$$

$$= 111.36$$

No axial thrust

$$F_1 = F_{r1} = 111.36 \text{ N}$$

$$L_{10} = \frac{60 \times 720 \times 8000}{10^6} =$$

$$C_1 = 1953.71 \text{ N}$$

$$B_1 \quad \boxed{6000}$$

$$R_{V2} = 232.4 \text{ N}$$

$$R_{H2} = 1261.2 \text{ N}$$

$$R_2 = \sqrt{R_{V2}^2 + R_{H2}^2}$$

$$= 1282.43 \text{ N}$$

$$F_{r1} = R_1$$

$$F_{r2} = R_2$$

$$F_2 = F_{r2} = 1282.43 \text{ N}$$

$$= 345.6 \text{ million rev.}$$

$$C_2 = 22499.09 \text{ N}$$

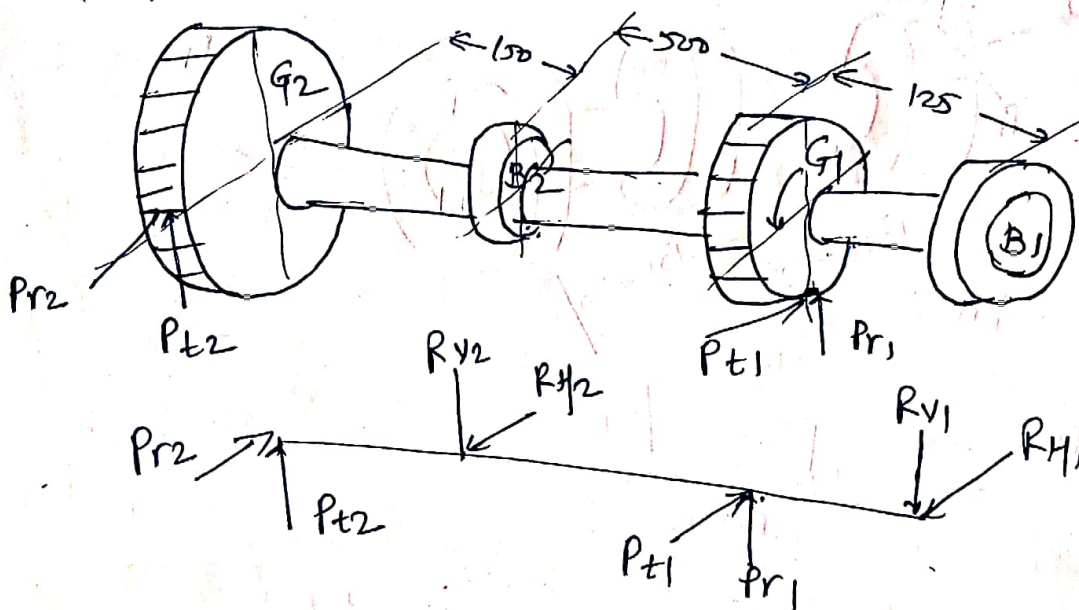
$$B_2 \quad \boxed{6404}$$

④ A shaft transmitting 50 kW at 125 rpm from the gear G_1 to the gear G_2 and mounted on two single row deep groove ball bearings B_1 & B_2 is shown in fig. The gear tooth forces are

$$P_{t1} = 15915 \text{ N} \quad P_{r1} = 5793 \text{ N}$$

$$P_{t2} = 9549 \text{ N} \quad P_{r2} = 3476 \text{ N}$$

The diameter of the shaft at bearings B_1 & B_2 is 75 mm. The load factor is 1.4 and the expected life for 90% of the bearings is 10 000 h. Select suitable ball bearings



$$R_{v1} = 2393 \text{ N}$$

$$R_{h1} = 11898 \text{ N}$$

$$F_{r1} = \sqrt{R_{v1}^2 + R_{h1}^2} = 12127 \text{ N}$$

$$F_{a1} = F_{a2} = 0$$

$$P_1 = F_{r1} = 12127$$

$$R_{v2} = 12999 \text{ N}$$

$$R_{h2} = 7493 \text{ N}$$

$$F_{r2} = 15004 \text{ N}$$

$$P_2 = F_{r2} = 15004 \text{ N}$$

$L_{10} = 75$ million revolution.

$$C_1 = 71598$$

$$C_2 = 88589 \text{ N}$$

$$\boxed{6315}$$

$$\boxed{6315}$$

⑤ A single-row deep groove ball bearing No. 6002 is subjected to an axial thrust of 1000 N and a radial load of 2200 N. Find the expected life that 50% of the bearings will complete under this condition.

→ Bearing 6002

$$G = 2500 \text{ N}$$

$$C = 5590 \text{ N}$$

$$F_r = 1000 \text{ N}$$

$$F_r = 2200 \text{ N}$$

$$\frac{F_a}{F_r} = 0.455$$

$$\frac{f_a}{e} = 0.4$$

$$\frac{f_a}{F_r} > e$$

$$Y = 1.2 - \frac{(1.2 - 1)}{(0.5 - 0.25)} (0.4 - 0.25) = 1.08$$

$$X = 0.56$$

Bearing life

$$P = X F_r + Y F_a = 2312 \text{ N}$$

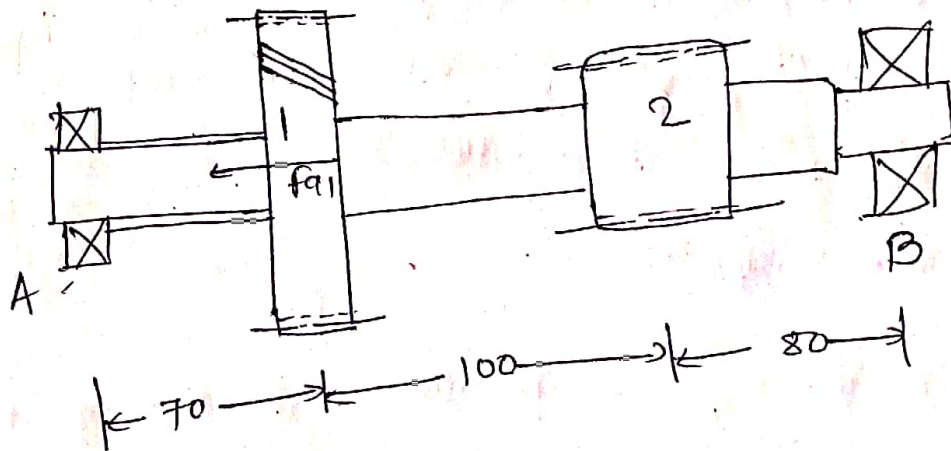
$$C = P (L_{10})^{1/3} = 49$$

$$L_{10} = 14.13 \text{ million rev.}$$

$$L_{50} = 5 L_{10} = 70.65 \text{ million rev.}$$

Example

- ⑥ An intermediate shaft of a two stage co-axial gear box, shown in fig, receives 10 kW power at 2880 rpm through right hand helix gear and transmit it to the output shaft through the spur pinion. The pitch circle diameters of helical gear and spur pinion are 450 mm and 108 mm respectively. The helix angle and normal pressure angle for helical gears are 23° & 20° respectively. The pressure angle for the spur pinion is 20° . The diameter of the shaft at bearings A and B are 50 mm & 60 mm respectively. The load factor is 1.8 & the expected rating life of the bearings is 25000 hr. select the deep groove ball bearings at A & B. The bearings are mounted such that bearing at A takes the thrust load.



* Design of cyclic load and speed:-

- In certain application, ball bearings are subjected to cyclic load and speed

- Suppose bearing is subjected to

- # F_1 radial load at N_1 speed for $\frac{T_1}{T}$ period 25% of time
- F_2 radial load at N_2 speed for $\frac{T_2}{T}$ period 50% of time
- F_3 radial load at N_3 speed for $\frac{T_3}{T}$ period 25% of time

- Under this circumstances, it is necessary to consider complete work cycle, for finding dynamic load capacity of the bearing.

- Divide complete work cycle into no. of parts based on operating conditions of load & speed.

- Suppose work cycle is divided into 'x' parts
 let $P_1, P_2, P_3, \dots, P_x$ be loads corresponding to speed $n_1, n_2, n_3, \dots, n_x$

- During first part L_1 is the life corresponding to load P_1 is given by

$$L_1 = (C/P_1)^3 \times 10^6 \text{ revolutions}$$

- In one revolution the life consumed is $(1/L_1)$

$$\left(\frac{1}{L_1}\right) = \frac{P_1^3}{C^3 (10^6)}$$

- During first part of rotation bearing undergo N_1 revolution. so life consumed by bearing during N_1 revolution is

$$N_1 \left(\frac{1}{L_1}\right) = \frac{N_1 P_1^3}{C^3 (10^6)}$$

- Similarly life consumed by bearing during 2nd part

$$\frac{N_2 P_2^3}{C^3 \times 10^6}$$

- The life consumed during complete work cycle can be written as

$$\frac{N_1 P_1^3}{C^3 \times 10^6} + \frac{N_2 P_2^3}{C^3 \times 10^6} + \frac{N_3 P_3^3}{C^3 \times 10^6} + \dots + \frac{N_x P_x^3}{C^3 \times 10^6} \quad (1)$$

- If P_e is the equivalent load for complete cycle (N) the life consumed by work cycle is given by

$$\frac{N P_e^3}{10^6 C^3}$$

$$\text{where } N = N_1 + N_2 + N_3 + \dots + N_x$$

$$\text{--- (2)}$$

Equating (1) & (2)

$$\frac{N_1 P_1^3}{C^3 \times 10^6} + \frac{N_2 P_2^3}{C^3 \times 10^6} + \dots + \frac{N_x P_x^3}{C^3 \times 10^6} = \frac{N P_e^3}{10^6 \times C^3}$$

$$N_1 P_1^3 + N_2 P_2^3 + \dots + N_x P_x^3 = N P_e^3$$

$$P_e = \sqrt[3]{\frac{N_1 P_1^3 + N_2 P_2^3 + \dots + N_x P_x^3}{N}}$$

$$P_e = \sqrt[3]{\frac{\sum N P^3}{\sum N}}$$

- When load does not vary in steps of const. magnitude, but varies continuously with time then

$$P_e = \left[\frac{\int_0^N P^3 dN}{\int_0^N dN} \right]^{1/3} = \left(\frac{1}{N} \int_0^N P^3 dN \right)^{1/3}$$

Example

① A single row deep groove ball bearing has a dynamic load capacity of 40500 N and operates on the following work cycle

- i) radial load of 5000 N at 500 rpm for 25% of the time.
 - ii) radial load of 10000 N at 700 rpm for 50% of time &
 - iii) radial load of 7000 N at 400 rpm for remaining time
- Calculate expected life of bearing in hours.

$C = 40500 \text{ N}$

consider work cycle of 1 min.

F_r	Time	rpm	rev. for
$F_{r1} = 5000$	0.25 time	500 rpm	125 (0.25 x 500)
$F_{r2} = 10000$	0.5 time	700 rpm	350
$F_{r3} = 7000$	0.25 time	400 rpm	100

$$P_e = \sqrt[3]{\frac{\sum N_i P_i^3}{\sum N_i}} = 8860.06 \text{ N}$$

$$L_0 = \left(\frac{C}{P_e}\right)^3 = 95.51 \text{ million rev.}$$

$$L_0 h = 2768.45 \text{ h.}$$

② A ball bearing is operating on a work cycle consisting of three parts - a radial load of 3000 N at 1440 rpm for one quarter cycle, a radial load of 5000 N at 720 rpm for one half cycle & radial load of 2500 N at 1440 rpm for remaining cycle. The expected life of bearing is 10000 h. Calculate the dynamic load carrying capacity of the bearing.

$N_1 = 360 \text{ rev. } N_2 = 360 \text{ rev. } N_3 = 360 \text{ rev.}$

$n = 1080 \text{ rpm.}$

$P_e = 3823, L_0 = 648 \text{ million rev. } C = 33082 \text{ N.}$

③ A single row deep groove ball bearing is subjected to a 30 sec. work cycle that consists of the following parts

	Part I	Part II
Duration (s)	10	20
Radial load (kN)	45	15
Axial load (kN)	12.5	6.25
Speed rpm	720	1440

The static & dynamic load capacities of the ball bearing are 50 & 68 kN resp. Calculate the expected life of bearing in hours.

→ Part I

$$f_a/f_r = 0.278 < e$$

$$f_a/c_0 = 0.25 \rightarrow e = 0.37$$

$$X_1 = 1 \quad Y_1 = 20$$

$$P_1 = f_r = 45000 \text{ N}$$

$$N_1 = \frac{10}{60} (720) = 120 \text{ rev.}$$

Part II

$$f_a/f_r = 0.417 > e$$

$$f_a/c_0 = 0.125 \rightarrow e = 0.31$$

$$X_2 = 0.56 \quad Y_2 = 1.42$$

$$P_2 = X_2 f_r + Y_2 f_a$$

$$= 17275 \text{ N}$$

$$N_2 = \frac{20}{60} (1440) = 480 \text{ rev.}$$

$$N_1 + N_2 = 600 \text{ rev.}$$

$$P_e = \sqrt{\frac{\sum N P^3}{\sum N}} = 28167.89 \text{ N}$$

$$L_{10} = (C/P_e)^3 = 14.069 \text{ million rev.}$$

$$L_{10} \text{ h} = 195.4 \text{ h}$$

* Bearing with a probability of survival other than 90% :-

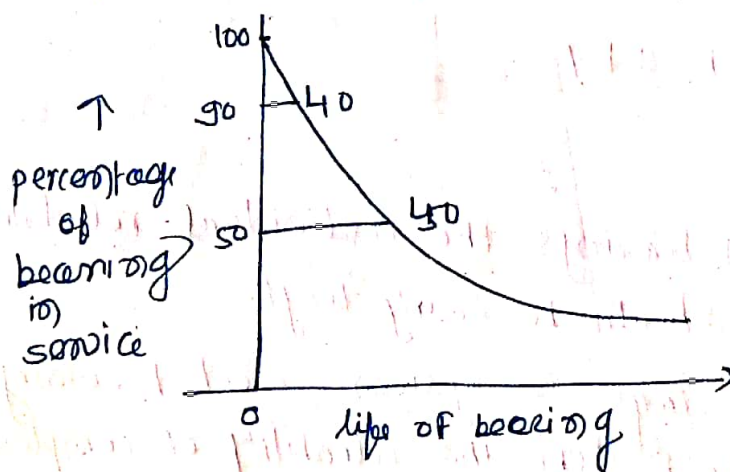
- Rating life is a life that 90% of group of bearing will complete before first fatigue crack failure

$$\text{Reliability } R = \frac{\text{No. of bearing which have successfully completed } L \text{ million revolutions}}{\text{Total no. of bearing under test}}$$

- Bearing selected based on manufacturer's catalogue have 0.9 or 90% reliability

- In some application, risk to human life is more so it becomes necessary to select bearing having more reliability.

- The relation betⁿ bearing life and reliability is given by weibull distribution.



for weibull distribution

$$R = e^{-\left(\frac{L}{a}\right)^b}$$

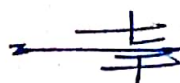
R = reliability
L = life corresponding to Reliability R (in fraction)

a & b = const

Rearranging weibull equation gives

$$\frac{1}{R} = e^{(L/a)^b}$$

$$\log_e\left(\frac{1}{R}\right) = \left(\frac{L}{a}\right)^b$$



————— (1)

Equation of Reliability & corresponding L_{10} life is written as

$$\log_e \left(\frac{1}{R_{90}} \right) = \left(\frac{L_{10}}{a} \right)^b \quad \text{--- (2)}$$

Dividing eqⁿ ① \div ② gives

$$\left(\frac{L}{L_{10}} \right) = \left[\frac{\log_e \left(\frac{1}{R} \right)}{\log_e \left(\frac{1}{R_{90}} \right)} \right]^{1/b} \quad \text{--- (3)}$$

where $R_{90} = 0.9$

& values of a & b are obtained from relation
 $L_{50} = 5 L_{10}$ --- (4)

substituting ④ in ③ gives

$$a = 6.84 \quad \& \quad b = 1.17$$

— If there are no. of bearings, the individual reliability of each bearing should be fairly high.

— If there are 'N' bearings in system, each having the same reliability R then the reliability of complete system is given by

$$R_s = (R)^N$$

R_s = probability of one out of N bearing failing during its lifetime.

Example:-

① A single row deep groove ball bearing is subjected to a radial force of 8 kN and thrust force of 3 kN. The values of X & Y factors are 0.56 & 1.5 resp. The shaft rotates at 1200 rpm. The diameter of the shaft is 75 mm and Bearing No. 6315 (C = 112000 N) is selected for this application.

- i) Estimate the life of this bearing, with 90% reliability
- ii) Estimate the reliability for 20000 h life

→ $F_r = 8 \text{ kN}$, $F_a = 3 \text{ kN}$ $X = 0.56$; $Y = 1.5$
 $n = 1200 \text{ rpm}$ $d = 75 \text{ mm}$
Bearing No. 6315 (C = 112000 N)

$$P = X F_r + Y F_a = 8980 \text{ N}$$

$$L_{10} = \left(\frac{C}{P} \right)^3 = 1940.10 \text{ million rev.}$$

$$L_{10h} = \frac{L_{10} (10^6)}{60n} = 26945.83 \text{ h}$$

Reliability for 20000 hr life

$$\left(\frac{L}{L_{10}} \right) = \left(\frac{\log_e (1/R)}{\log_e (1/R_{90})} \right)^{1/b}$$

$$R = 0.9283 = 92.83 \%$$

② A ball bearing, subjected to a radial load of 5 kN is expected to have a life of 8000 h at 1450 rpm with a reliability of 99%. Calculate the dynamic load capacity of bearing, so that it can be selected from the manufacturer's catalogue based on 90% reliability.

$$F_r = 5 \text{ kN}$$

$$n = 1450 \text{ rpm}$$

$$L_{10h} = 8000 \text{ h.}$$

Bearing life for 90% reliability

$$L_{90} = \frac{60 n L_{10h}}{10^6} = 696 \text{ million rev.}$$

Bearing life with 90% reliability

$$\left(\frac{L_{90}}{L_{10}} \right) = \left[\frac{\log_e(1/R_{90})}{\log_e(1/R_{10})} \right]^{1/1.17}$$

$$L_{10} = 5786.29 \text{ million rev.}$$

Dynamic load

$$(C/P)^3 = L_{10}$$

$$[C = 86547.7 \text{ N}]$$

③ A single-row deep groove ball bearing is used to support the lay shaft of a four speed automobile gearbox. It is subjected to the following loads in respective speed ratios:-

gear	axial load (N)	radial load (N)	% time engaged
first gear	3250	4000	1%
second gear	500	2750	3%
third gear	50	2750	21%
fourth gear	nil	nil	75%

The lay shaft is fixed to the engine shaft & rotates at 1750 rpm. The static & dynamic load carrying capacities of the bearings are 11600 and 17600 N resp. The bearing is expected to be in use for 4000 hrs of operation. find out the reliability with which the life could be expected.

→ $n = 1750 \text{ rpm}$
 $C_0 = 11600$
 $C = 17600 \text{ N}$
 $L_h = 4000 \text{ hrs.}$

- Equivalent load for complete work cycle by considering one minute duration

$$N_1 = \frac{60}{100} (1750) = 17.50 \text{ rev.}$$

$$N_2 = 52.50 \text{ rev}$$

$$N_3 = 367.50 \text{ rev}$$

$$N_4 = 1312.50 \text{ rev.}$$

$$N = N_1 + N_2 + N_3 + N_4 = 1750 \text{ rev.}$$

for 1st Gear

$$P_1 = X F_r + Y F_a$$

$$(F_a/F_r) = 0.8125 \quad (F_a/C_0) = 0.28 \quad e = 0.37 - 0.44$$

$$F_a/F_r > e \quad X = 0.56 \quad Y = 1.176$$

$$P_1 = X F_r + Y F_a = 6062 \text{ N}$$

for 2nd Gear

$$(F_a/F_r) = 0.182 \quad (F_a/C_0) = 0.0431 \quad e = 0.24 - 0.27$$

$$(F_a/F_r) < e$$

$$P_2 = F_r = 2750 \text{ N}$$

for 3rd Gear. $P_3 = F_r = 2750 \text{ N}$

for 4th Gear $P_4 = 0$

$$P_e = \sqrt[3]{\frac{\sum P_i^3}{\sum N}} = 1932.67 \text{ N}$$

$$L_0 = (C/P)^3 = 755.2 \text{ million rev.}$$

$$L = \frac{60000 L_0}{10^6} = 420 \text{ million rev.}$$

Reliability

$$\left(\frac{L}{L_0}\right) = \left[\frac{\log_e(1/R)}{\log_e(1/R_0)}\right]^{1/b} \quad b = 1.17$$

$$R = 0.9483 = 94.83 \%$$